Human Capital and Savings in an OLG Economy with Migration Possibilities: A Theoretical Note

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In a theoretical study by Stark, Helmenstein, and Prskawetz (1998), the presence of migration possibilities may increase human capital formation. This note verifies the robustness of the said result by introducing savings as a choice variable in an overlapping generations model with migration possibilities. Results indicate that human capital formation will still increase provided that certain technical conditions are met.

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In an article on brain drain, Stark, Helmenstein, and Prskawetz (1998) showed that human capital formation could improve even before migration is realized as long as there is a positive probability of securing employment in a foreign country. The said study proposed an overlapping generations (OLG) model wherein the representative individual will work in the home country in period 1 but has the opportunity to migrate in period 2 after accumulating human capital. In this case, migration acts as a positive inducement device for increasing human capital accumulation in the home country as long as there is a positive probability of securing employment in the foreign country. Furthermore, they showed that even when the home country subsidizes human capital accumulation, the results remain invariant. This apparently addresses a key robustness requirement since some countries do provide educational subsidies that enhance human capital accumulation.

The model, however, relies on the assumption that workers spend all earnings in both periods, thereby ignoring the role of saving behaviour in human capital accumulation.

In this minor theoretical note, we introduce savings in the model of Stark et al. (1998). The consideration of the decision to save reflects, to a larger extent, key mechanisms that support the migration decision. There are some reasons why introducing savings may be important. First, in the empirical literature, migration is not costless. In this regard, savings may be generated to finance migration. Second, by including savings, agents face additional trade–offs. While an increase in the probability of employment will increase human capital, it may result in a reduction in savings which may have implications on physical capital accumulation. We show, though, that the inclusion of savings as a choice variable in this OLG economy imposes a technical requirement for human capital accumulation to respond positively to changes in the probability of gaining a job abroad.

The note is organized as follows: Section II discusses the model and key results. The last section concludes.

SAVINGS AND MIGRATION OPPORTUNITIES

Following the modeling strategy of Stark et al. (1998), consider a two period OLG model with perfect foresight. Let the labor endowment be denoted by In the first period, individuals decide on how much labor endowment to allocate to human capital activity or to working. In the second period the individual observes a positive probability of securing a job in the foreign country, an indication that migration opportunities are present. Let the utility function, be strictly concave in its arguments and time separable, which implies diminishing marginal utility of consumption in both time periods.represents the discount factor applied to the subutility in the second period. Consider the case that a proportion of first period wage income is saved. At time t, assume thatlabor income is allocated to savings and consumption, that is, , where is the proportion of labor endowment that is allocated to work and is the amount of savings.[1] Thus, consumption is a negative function of savings and human capital investments.

Following Stark et al. (1998), a strictly concave production function for human capital exist, converting units of labor endowment allocated to human capital in the first period to realized human capital in the next period. We assume that the usual properties of production functions hold.

Consider what happens in the second period. If we assume that there are no migration possibilities, then consumption in this period will be determined domestically. Let c_{t+1} be the consumption of the representative agent which is equal to $(1 + r_{t+1})s_t + \varphi(\ell)w = (1 + r_{t+1})[(1 - \ell)w - c_t] + \varphi(\ell)w$. It is clear then that when $s_t = 0$, $c_{t+1} = \varphi(\ell)w$, hence we go back to the original formulation of Stark et al. (1998). It is obvious that the individual has higher consumption possibilities in period 2. We assume that savings grows by the rate of interest in period t+1. Following Stark et al. (1998), migration opportunity is present when $w < w^f$, that is, when the competitive wage in foreign country f is greater than that of the home country's in period t+1. Thus, we can define $c^f_{t+1} = (1 + r_{t+1})s_t + \varphi(\ell)w^f$.

Consider the probability weighted utility (expected utility) in period 2. Let be the probability that a job can be secured in the foreign country.

$$\pi U\left[(1 + r_{t+1})s_t + \varphi(\ell)w^f\right] + (1 - \pi) U[(1 + r_{t+1})s_t + \varphi(\ell)w] \quad (1)$$

Combining the above representation of period 2 utility with that of the first period, the lifetime utility of the representative agent is

$$U[(1 - \ell)w - s_t] + \beta\{\pi U\left[(1 + r_{t+1})s_t + \varphi(\ell)w^f\right] + (1 - \pi)U[(1 + r_{t+1})s_t + \varphi(\ell)w]\} \quad (2)$$

Since the lifetime utility function is a linear combination of strictly concave function where β, π, and $(1 - \pi)$ are all nonnegative, we note that the lifetime utility function is also strictly concave. Maximizing (2) with respect to s_t and ℓ_t and subject to the open convex constraint of $\ell \in (0,1)$, we have the two first order conditions (FOCs):

$$-U'(c_t) + (1 + r_{t+1})\beta\left[\pi U'\left(c^f_{t+1}\right) + (1 - \pi)U'(c_{t+1})\right] = 0 \tag{3}$$

$$-U'(c_t)w + \varphi'(\ell)\beta\left[\pi U'\left(c^f_{t+1}\right)w^f + (1 - \pi)U'(c_{t+1})w\right] = 0 \tag{4}$$

Rewriting (3) and (4), we have

$$\frac{U'(c_t)}{\beta\{\pi[U'(c^f_{t+1}) - U'(c_{t+1})] + U'(c_{t+1})\}} = (1 + r_{t+1}) \tag{3'}$$

$$\frac{U'(c_t)}{\beta\left\{\pi\left[U'\left(c^f_{t+1}\right)\frac{w^f}{w} - U'(c_{t+1})\right] + U'(c_{t+1})\right\}} = \varphi'(\ell) \tag{4'}$$

Equation (4') corresponds to Stark et al.'s (1998) main result when $s_t = 0$.

Following Stark et al. (1998), we can express equations (3') and (4') as f^1: $\psi_s(\ell, \pi, s, r) - (1 + r_{t+1}) = 0$ and f^2: $\psi_\ell(\ell, \pi, s, r) - \varphi'(\ell) = 0$ or $\mathbf{f}(\ell, \pi, s, r) = \mathbf{0}$. We look into the critical requirement for the existence of implicit functions. Define matrix J as

$$J = \begin{bmatrix} \frac{\partial\psi_s(\ell,\pi,s,r)}{\partial s} & \frac{\partial\psi_s(\ell,\pi,s,r)}{\partial \ell} \\ \frac{\partial\psi_\ell(\ell,\pi,s,r)}{\partial s} & \frac{\partial\psi_\ell(\ell,\pi,s,r)}{\partial \ell} - \varphi''(\ell) \end{bmatrix}$$

which is also the Hessian for the second order condition (SOC) of the maximization problem. Since the lifetime utility function is strictly concave we know that there exist a unique $\bar{\ell} = \bar{\ell}(\pi, r)$ and $\bar{s} = \bar{s}(\pi, r)$ that solves $\mathbf{f}(\ell, \pi, s, r) = \mathbf{0}$. We also observe that the Hessian is negative definite and therefore nonsingular, hence the implicit function theorem is applicable.

Thus, we have the following proposition:

Proposition. In an OLG economy with migration possibilities, the incorporation of the decision to save will reinforce the beneficial effects of migration on human capital accumulation.

Proof. Assume that only π will change. By implicit differentiation of the first order conditions with respect to π, we have the following matrix equation:

$$J\boldsymbol{z} = -\boldsymbol{b}$$

where $\boldsymbol{z} = \left[\frac{ds}{d\pi} \; \frac{d\ell}{d\pi}\right]$ and $-\boldsymbol{b} = -\left[\frac{\partial\phi_s(\ell,\pi,s,r)}{\partial\pi} \frac{\partial\phi_\ell(\ell,\pi,s,r)}{\partial\pi}\right]'$. Define J_1 as the resultant matrix when we replace the first column of J by $\boldsymbol{b}$. Since J is negative definite, this implies that $|J|$ is positive. By assumption, $|J_1| > 0$ or $-\frac{\partial\phi_s(\ell,\pi,s,r)}{\partial s}\frac{\partial\phi_\ell(\ell,\pi,s,r)}{\partial\pi} + \frac{\partial\phi_\ell(\ell,\pi,s,r)}{\partial s}\frac{\partial\phi_s(\ell,\pi,s,r)}{\partial\pi} > 0$, rearranging the terms we have:

$$\frac{\frac{\partial\psi_s(\ell,\pi,s,r)}{\partial s}}{\frac{\partial\psi_\ell(\ell,\pi,s,r)}{\partial s}} \cdot \frac{\frac{\partial\psi_\ell(\ell,\pi,s,r)}{\partial\pi}}{\frac{\partial\psi_s(\ell,\pi,s,r)}{\partial\pi}} < 1$$

where:

$$\frac{\partial\psi_\ell(\ell,\pi,s,r)}{\partial s} = \frac{-U''(c_t)\left\{\pi\left[U'\left(c^f_{t+1}\right)\frac{w^f}{w} - U'(c_{t+1})\right] + U'(c_{t+1})\right\} - (1 + r_{t+1})\left\{\pi\left[U''\left(c^f_{t+1}\right)\frac{w^f}{w} - U''(c_{t+1})\right] + U''(c_{t+1})\right\}U'(c_t)}{\beta\left\{\pi\left[U'\left(c^f_{t+1}\right)\frac{w^f}{w} - U'(c_{t+1})\right] + U'(c_{t+1})\right\}^2} \tag{5}$$

$$\frac{\partial\psi_s(\ell,\pi,s,r)}{\partial s} = \frac{-U''(c_t)\{\pi[U'(c^f_{t+1}) - U'(c_{t+1})] + U'(c_{t+1})\} - (1 + r_{t+1})\{\pi[U''(c^f_{t+1}) - U''(c_{t+1})] + U''(c_{t+1})\}U'(c_t)}{\beta\{\pi[U'(c^f_{t+1}) - U'(c_{t+1})] + U'(c_{t+1})\}^2} \tag{6}$$

$$\frac{\partial\psi_s(\ell,\pi,s,r)}{\partial\pi} = -\frac{U'(c_t)}{\beta[\pi U'(c^f_{t+1}) + (1-\pi)U'(c_{t+1})]^2}\left[U'(c^f_{t+1}) - U'(c_{t+1})\right] \tag{7}$$

$$\frac{\partial\psi_\ell(\ell,\pi,s,r)}{\partial\pi} = -\frac{U'(c_t)}{\beta\left[\pi U'\left(c^f_{t+1}\right)\frac{w^f}{w} + (1-\pi)U'(c_{t+1})\right]^2}\left[U'\left(c^f_{t+1}\right)\frac{w^f}{w} - U'(c_{t+1})\right] \tag{8}$$

Evaluating each term where $\frac{\partial\phi_\ell(\ell,\pi,s,r)}{\partial s} \cdot \frac{\partial\phi_s(\ell,\pi,s,r)}{\partial\pi} > 0$ and simplifying we have:

$$\frac{U''(c_t)}{U'(c_t)} > (1 + r_{t+1})\left[\pi\frac{U''(c^f_{t+1})}{U'(c^f_{t+1})} + (1 - \pi)\frac{U''(c_{t+1})}{U'(c_{t+1})}\right] \tag{9}$$

Equation (9) represents a key condition, showing that for the decision to save to reinforce human capital accumulation, the curvature of the utility function at current consumption should be greater than the probability weighted sum of the respective curvatures of the utility function at c^f_{t+1} and c_{t+1}. Thus, $\frac{d\ell}{d\pi} > 0$ can be established.

CONCLUDING REMARKS

This note has attempted to incorporate savings into a closed OLG economy whose residents face migration possibilities. The paper was able to show that the key result in the reference paper which is the positive response of human capital to changes in the probability of employment remains robust despite the inclusion of the decision to save into the model. This may be expected since a positive relationship between human capital accumulation and the probability of getting employed implies that as the amount of endowment devoted to human capital accumulation increases, the proportion of labor endowment allocated to work falls, thereby affecting the level of savings. However, we explicitly did not introduce savings motives and capital accumulation, the inclusion of the latter may highlight the tension between human capital and physical capital accumulation.

NOTE

[1] This is typical of OLG models wherein physical capital is determined during the time the generation is young. This means that the higher is the lower will be the first period consumption and savings but second period consumption will be higher.

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