

RESEARCH ARTICLE

On the Existence of Calendar Anomalies and Persistence in the Daily Returns of the PSEi

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Abstract: The future of the stock market may never be predicted consistently, nor its past behavior understood entirely, but any knowledge gained from observing it could help decide on a sound investment strategy. In this study, I looked at the daily returns of the Philippine Stock Exchange index (PSEi) from March 1, 1990, to January 31, 2017, and see how the data relates to the mathematically verifiable aspects of the noise theory and efficient market theory (EMT). In relation to the noise theory, I looked at the occurrences of anomalies. For the EMT, I made use of discrete-time Markov chains to determine some trends. The study results showed that most stock market anomalies are present while persistent behavior is hardly present in the dataset. Furthermore, I applied day ahead time domain forecasting methods starting with the simple moving average models to autoregressive moving average models. The augmented Dickey-Fuller test indicate that the daily returns are a stationary series although the ACF and PACF plots have consistently shown non-zero correlations for lags 1, 9, 12, 13. I have obtained AR(1) and ARMA(1,2) processes for the data and both models indicate the same forecasting accuracy via the Diebold-Mariano test. Although these time domain processes were unable to predict the random noise in the data, these processes were accurate in predicting the signs of the values as supported by the Pesaran-Timmermann test.

Keywords: PSE index, Markov chains, stock market anomalies, long memory

JEL Classifications: G10, G14

History is our only source of hard statistics (Bogle, 2010, p. 258). Although in the case of the stock market, the past is not always predictive of the future. Any information that becomes available to investors has the potential of changing their investing behavior immediately; profitable information could be immediately acted upon while negative news could just as easily drive investors out. Nevertheless, investors should heed the advice of Malkiel (1999, p. 278): “*Even if stock prices move randomly, you shouldn’t.*”

The Philippine Stock Exchange (PSE) came about when the Manila Stock Exchange and Makati Stock Exchange were unified on December 23, 1992 (“About PSE,” n.d.). The Philippine Stock Exchange index (PSEi) is the health barometer of the Philippine stock market. It consists of the 30 largest listed companies based on the public float (the portion of a company’s shares owned by public investors), liquidity (the ease of owning and selling without having change in the price), and market capitalization (product of the company’s

outstanding shares and the stock price per share). For a company to qualify, it must have at least a public float of 12%, it must be in the top quartile based on the median daily value per month for at least nine out of 12 months, and it must be in the top 30 based on the full market capitalization (Dumlao-Abadilla, 2017).

According to the PSE (2016), in 2015, the number of stock market accounts has experienced a 46.1% compounded annual growth since 2010. As of 2015, the total number of market accounts is 712,549 with 98.5% belonging to local investors. Among the total accounts, 95.2% are retail accounts while the rest are institutional accounts. Although the number of accounts has been growing, the numbers indicate that less than 1% of the population invests directly in the stock market, but institutional accounts such as mutual funds or pensions may also include a good number of local investors.

Investing in stocks, real estate, and small business is believed to be the best way of building wealth (Tyson, 2011). Among the three, investing in stocks is the easiest investment vehicle to get into. One can start investing in the PSE for as little as PhP5,000 (US\$95) through an online broker; and with the launching of the Personal Equity and Retirement Account (Agcaoili, 2016), some index funds that track the PSEi can be had for as little as PhP1,000 (US\$19).

In this paper, I looked at the behavior of the stock market through an analysis of the daily returns obtained from the closing prices of the PSEi from February 28, 1990, up to January 31, 2017. I looked at the behavior in relation to mathematically verifiable aspects of the noise theory and efficient market theory (EMT) such as the existence of several anomalies and the correlation structure of the process.

Although most studies or theories on the stock market were based on what happened in the Wall Street, which is the home of the New York Stock Exchange (NYSE) and the Nasdaq Stock Exchange (NASDAQ), it would be interesting to see if these observations translate to the local stock market. The Standard and Poor 500 (S&P 500) index is considered a good representation of equities with a large market capitalization listed as common stocks in the NYSE or NASDAQ. The correlation between the annual returns of the S&P 500 index and PSEi from 1991 to 2016 is 0.363242 which indicates a positive but weak linear relationship.

PSEi Data

PSE provided the dataset of closing prices. From the daily closing prices, I determined the daily rate of return as

$$i_k = \frac{x_k - x_{k-1}}{x_{k-1}}, \quad (1)$$

where $k = 2, 3, \dots$, and x_k denotes the closing price of the k th day. I studied a time series of daily returns with 6,641 data points that started on March 1, 1990, until January 31, 2017. The average daily return and standard deviation of the dataset are 0.0403% and 1.4769%, respectively. The coefficient of variation (ratio between the standard deviation and mean) is 3.6452, which translates to high variability in relation to the mean.

From the daily returns, the effective rate of return over k days can be computed as

$$\prod_k (1 + i_k) - 1. \quad (2)$$

Kellison (1991) defined the effective rate of return as “the amount of money that one unit invested at the beginning of a period will earn during the period, where interest is paid at the end of the period” (p. 4). Taking the effective rate of return enables the comparison of the interests earned over the same length of time.

If the interests are to be compared over different time horizons (total length of time an investor holds the investment), the compound annual growth rate (CAGR) can be computed, which is the corresponding nominal rate when interest is compounded annually. Having n years with k trading days, the corresponding compound annual growth rate over those years is

$$\sqrt[n]{\prod_k (1 + i_k)} - 1. \quad (3)$$

Note that results of Equations (1), (2), and (3) are the decimal representation of the rate of return, so the result has to be multiplied by 100% to obtain the rate of return.

I determined the CAGR of investment horizons of length n ($= 1, 2, \dots, 20$) over 1991 to 2016. For each n , I computed the CAGR over the intervals 1991 to 1991 + n - 1, 1992 to 1992 + n - 1, until 2016 - n + 1 to 2016. Observe that as the length of the time horizon increases, the number of intervals decreases.

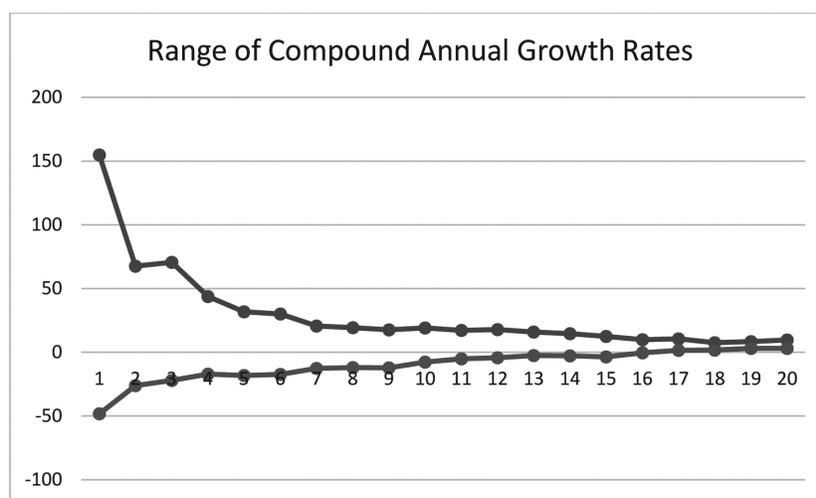


Figure 1. Historically, an investment horizon of at least 17 years guarantees that all the CAGRs are positive.

In general, for each n , the sample size is $26 - (n - 1)$. So for a time horizon of, say, five years, I obtained the CAGR of the 22 time intervals 1991-1995, 1992-1996, 1993-1997, until 2012-2016.

Figure 1 shows the range (the difference between the maximum value and minimum value) of the CAGR for each investment horizon. For an investment horizon of 1, the minimum value is -48.2867% and the maximum value is 154.7831% but staying a year more reduces the range immensely with a minimum of -26.1708% and a maximum of 67.58928%. In general, as n increases, the range decreases which indicates a decrease in the dispersion or variability.

Other measures of variation also support the decreasing variability or dispersion as shown by the range. The standard deviation also decreased from 40.96643 to 1.830528 as increased from one year to 18 years; for 19 and 20 years, the standard deviations are 1.848485 and 2.369687, respectively. In a dataset of investment returns, the reciprocal of the coefficient of variation can be thought of as the Sharpe ratio which is the measure of the risk-adjusted return for each investment horizon. The Sharpe ratio is the annual rate of return (above the risk-free rate of return on the U.S. Treasury bills) per unit of risk as measured by the standard deviation (Bogle, 2010, p. 96). I can argue that there is no such thing as a risk-free investment and use the average CAGR for the average annual rate of return. In an investment horizon of 1–19 years, the Sharpe ratio increased from 0.381119 to 3.122225; and at an investment horizon of 20, the Sharpe ratio is

2.496877. This implies that, historically, an investment horizon of 19 gives the best return per unit of risk added.

Noise Theory

Graham and Zweig (2005) introduced the parable of Mr. Market—he is an obliging business partner who keeps the investor updated with the value of their investment on a daily basis. Mr. Market sometimes offers to buy or sell whatever the investor is interested in, and although he manages to occasionally give a fair value, there are instances when his enthusiasm or fear gets the best of him, resulting to prices that are either very high or very low. Graham likened the behavior of the stock market to Mr. Market's manic depression. In the noise theory model, trading is conducted by ill-informed investors who act on sentiment rather than rational thinking (Cunningham, 2001, p. 26). Examples of such behaviors are acting on tips and rumors, high portfolio turnover rates, buying high and selling low, and paying excessive management fees for poorly managed funds (Cunningham, 2001, p. 27).

Cunningham (2001) used Mr. Market's bipolar disorder to explain the occurrence of bursts (daily rates of return exceeding 3%) or busts (daily drops exceeding 3%) and anomalies. The largest daily rate of return of the PSEi is 17.55% which occurred on January 22, 2001. This is the first trading day after Gloria Macapagal-Arroyo was sworn in as the 14th president of the Republic on Jan 20, 2001, which was

a Saturday (“2001 in the Philippines,” n.d.). That burst was market enthusiasm over the promise of a stable leadership after the 2nd EDSA Revolution which removed Joseph E. Estrada from the Office of the President of the Philippines. Incidentally, the second largest daily rate of return of 16.48% occurred on November 6, 2000, which is a week before the start of the impeachment trial of Joseph E. Estrada that started on November 13, 2000, until January 17, 2001 (“2000 in the Philippines,” n.d.). The largest daily drop of the PSEi is -12.26% which occurred on October 27, 2008. Most global stock markets plunge by more than 10% on October 24, 2008 (Kumar, 2008) and October 27, 2008, is the first trading day after the global market meltdown.

Calendar Effect

Most unexplained stock market phenomena is related to the calendar such as the weekend effect (prices are generally lower on Mondays and higher on Fridays), month effect (prices rise at the end and the beginning of months), and January effect (prices rise in January; Cunningham, 2001 p. 11). By examining the returns, the anomalies are also observable in the PSEi data.

The January effect is also known as the turn-of-the-year effect, and some investors even use this as an investing strategy in which entry to the market is made during the last week of December until the first two

weeks of January. I computed for the effective rate of return at the turn of the year using Equation (2) over the last five trading days of December to the first 10 trading days of January.

The compound rates of return at the turn of the year varied from -11.4945% to 13.5153%. Out of the 26 years in the dataset, the effective rate of return at the turn of the year was only negative five times; and at the turn of the 21st century, negative returns were experienced twice (2008 and 2016). The average effective rate of return at the turn of the year is only 3.2712%. Nevertheless, this strategy shows that when carried out consistently, it could be profitable. See Figure 2 for a comprehensive summary of the turn of the year compound returns.

Furthermore, I also looked at the average effective rate of return for each month using Equation (2) where varies over the number of trading days in a month. Figure 3 shows that December and January have the best average effective rates of return. The worst average of -3.5898% is obtained during August which is traditionally part of the Ghost Month Festival. This month is believed to bring bad luck and some activities are to be avoided to prevent it; opening a business or signing contracts are a few of such activities (Requintina, 2016).

At the turn of the month, it is also observed that prices tend to rise. I checked its presence in the PSEi by, initially, looking at the average return for each day

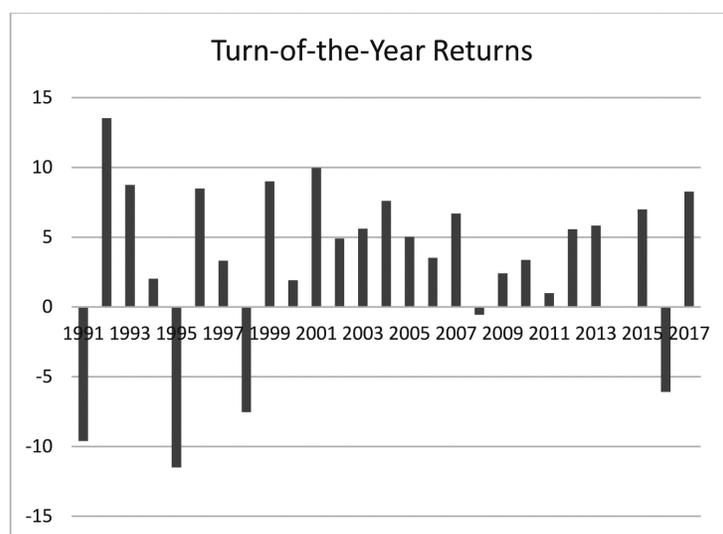


Figure 2. The largest compound rate of return over the last five trading days of December to the first 10 trading days of January of the indicated year is 13.5153% which was obtained at the turn to the year 1992 while the largest drop is -11.4945% which was obtained at the turn to the year 1995.

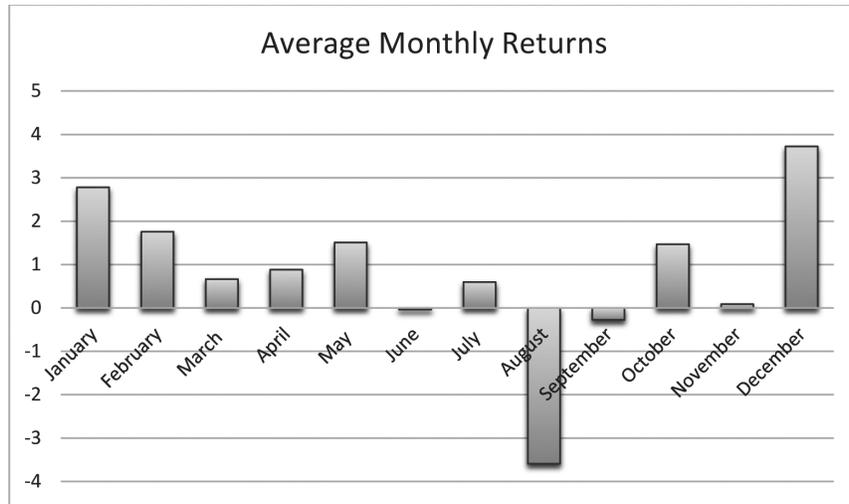


Figure 3. The best month to invest in the market is in December followed by January which shows an average monthly effective rate of 3.7217% and 2.7785%, respectively.

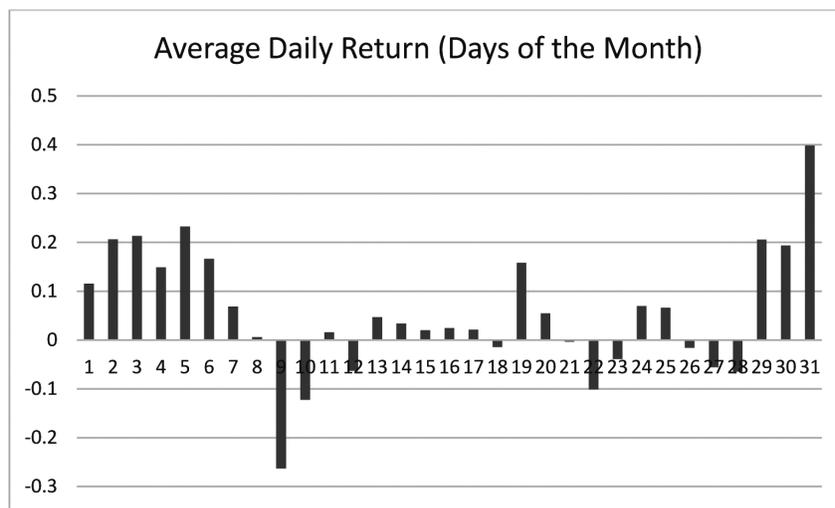


Figure 4. The average daily return rate on the last three days of the month are all positive with the last day having twice as much return compared to practically any day of the month. On average, the rates of return at the start of the month are increasing on the first three days.

of the month. The highest average rate of return is 0.3985% which occurred on the 31st. Figure 4 shows that most of the positive average returns are clustered at the start and the end of the month.

Because of this clustering, I computed for the average effective rate of return that covers the last three trading days of the previous month to the first three trading days of the current month. I applied Equation (2) over the six days mentioned above to compute for the average rate of return at the turn of the month. There are 322 turn-of-month in the dataset and the average effective rate of return is 0.9446%. Not

all months have its first trading day on the 1st so the first three trading days may cover up to the 5th which, incidentally, has the second highest average return rate of 0.2327%. The average return rate declines after the 5th and behaves flatly until practically at the end of the month.

At the turn of the week, it has been observed that there is euphoria on Fridays and apprehensions on Mondays. I checked its existence on the data by taking the average percent change of each trading day of the week. As shown in Figure 5, this behavior is apparent, although Tuesday has the worst average return rate of

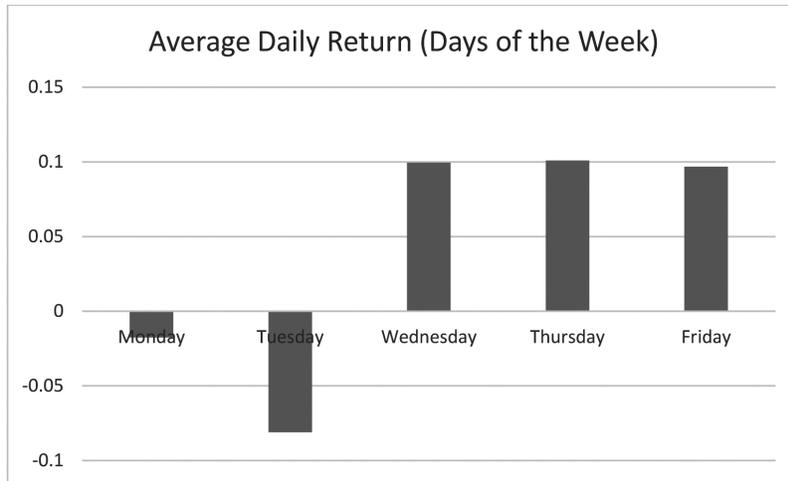


Figure 5. Taking the averages of the daily rates of return for each weekday indicates that although Tuesday has, historically, the worst daily return the stock market still tends to go down on Mondays.

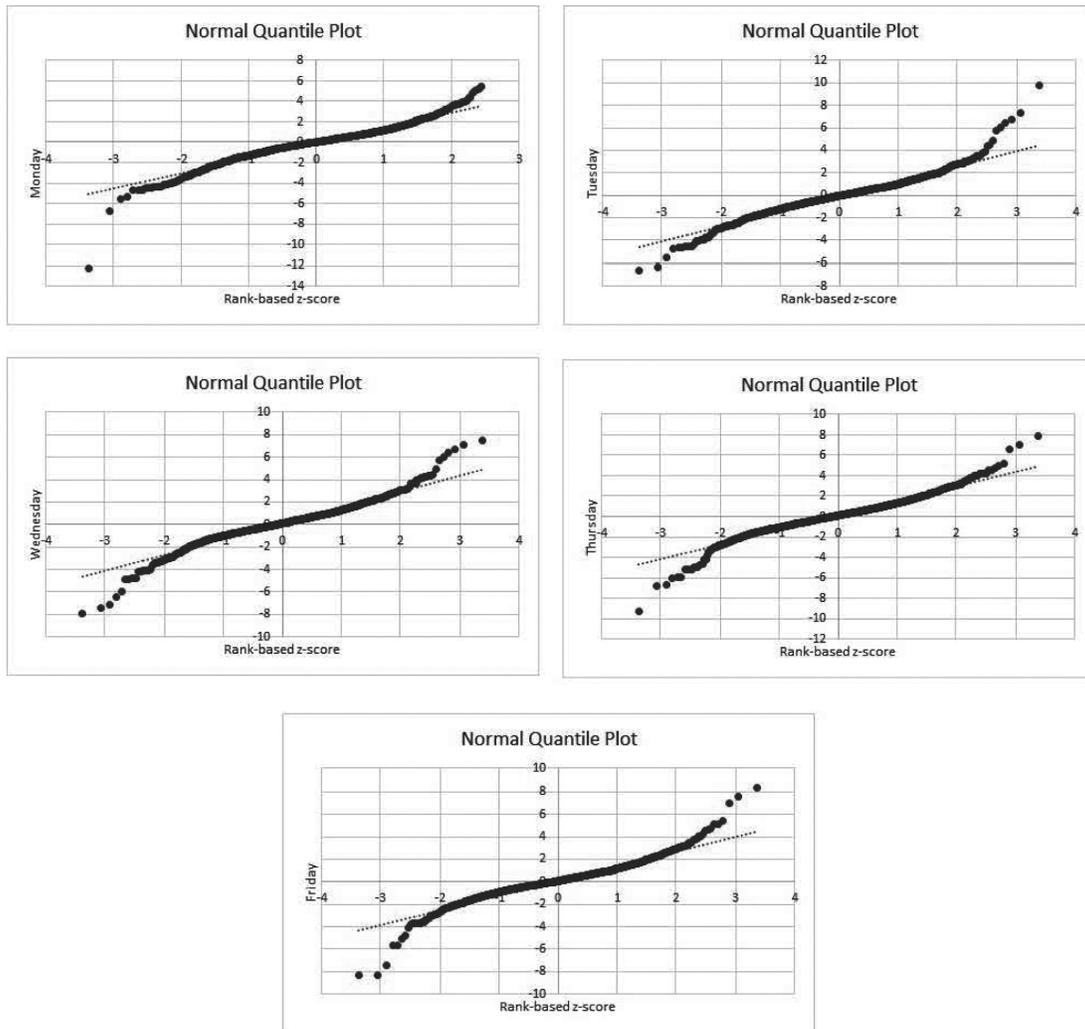


Figure 6. The normal quantile plot for the returns of each day generally falls on a straight line except for the extreme values, which indicates that the datasets can be assumed to be normally distributed.

-0.0810% compared to Monday which has a negative average return rate too of -0.0175% while Thursday has the best average return rate of 0.1010%. The averages for the last three days of the week are so close to each other that these values may not even be statistically different. The averages on Wednesday, Thursday, and Friday are all at least twice as much as the average daily return rate of the entire dataset which, as mentioned earlier, is 0.0403%.

I performed a one-way analysis of variance (ANOVA) to confirm that the averages for each day are statistically not all equal. An ANOVA works under the assumption that the datasets are normally distributed, so I generated the normal quantile plot of the returns for each day. Figure 6 shows that the normal quantile plots generally follow a straight line except for some small and large values. This indicates that it is safe to assume that the datasets come from a normal distribution.

Tables 1 and 2 show the results from the single-factor ANOVA. The p -value for the ANOVA is 0.001612 which indicates that at the significance level of $\alpha = 0.05$ the null hypothesis that the averages of the returns are all equal is rejected. This result supports the alternative hypothesis that the averages are not all equal. Recall that if the p -value is less than or equal to α , the null hypothesis is rejected and the sample is said to support the alternative hypothesis.

I performed a pairwise analysis on the averages to determine which pairs are statistically different via a two-tail two-sample t -test for equal means. Before I performed the t -test for each pair, a two-sample F -test for variances to each pair must be run to determine whether the variances are statistically equal or not. The null hypothesis for this test is that the variances are equal while the alternative hypothesis is that the variances are unequal. In Table 1, the values of the variances for the pairs Wednesday/Thursday and Tuesday/Friday are quite close, and the F -test for these pairs fail to reject the null hypothesis at the significance level of $\alpha = 0.05$ as shown in Table 3. This means that for these pairs, the two-sample t -test can be applied, assuming equal variances while using the two-sample t -test for assuming equal variances for the rest of the pairs.

I applied the appropriate two-sample t -test for equal means based on the result of the two-sample F -test for variances. The null hypothesis for this test is that the means of the returns for each day are equal with the alternative hypothesis that the means are unequal. Table 4 contains the p -values for the two-sample t -test for equal means and the results show that, except for the pairs Tuesday/Wednesday, Tuesday/Thursday, and Tuesday/Friday, the means for each pair of days are statistically equal.

Because of the result of the two-sample t -test for equal means, I performed an ANOVA involving

Table 1. Results of F -test for Variances

SUMMARY				
Groups	Count	Sum	Average	Variance
Monday	1289	-16.6097	-0.01289	2.997435
Tuesday	1358	-116.243	-0.0856	1.895877
Wednesday	1360	135.9542	0.099966	2.110462
Thursday	1337	138.2081	0.103372	2.113121
Friday	1297	131.3079	0.10124	1.841276

Table 2. ANOVA Test Results

ANOVA						
Source of Variation	SS	df	MS	F	p-value	F crit
Between Groups	40.14446	4	10.03611	4.589616	0.001612	2.373271
Within Groups	14510.94	6636	2.1867			
Total	14551.09	6640				

Table 3. *Results of Two-Tailed, Two-Sample F-Test*

	Monday	Tuesday	Wednesday	Thursday	Friday
Monday		4.96E-17	9.29E-11	1.31E-10	1.45E-18
Tuesday	4.96E-17		0.024145	0.023301	0.297557
Wednesday	9.29E-11	0.024145		0.490719	0.006535
Thursday	1.31E-10	0.023301	0.490719		0.0063
Friday	1.45E-18	0.297557	0.006535	0.0063	

Table 4. *Two-Sample T-Test Results*

	Monday	Tuesday	Wednesday	Thursday	Friday
Monday		0.233399	0.070046	0.062973	0.062316
Tuesday	0.233399		0.000641	0.000541	0.000439
Wednesday	0.070046	0.000641		0.951485	0.981364
Thursday	0.062973	0.000541	0.951485		0.968953
Friday	0.062316	0.000439	0.981364	0.968953	

Monday, Wednesday, Thursday, and Friday; the resulting p -value is 0.130437 which indicates that at the significance level of $\alpha = 0.05$ the null hypothesis that all the averages are equal is accepted.

In some stock markets, it is observed that the returns on Mondays and Fridays are significantly different from the other days where returns on Mondays are significantly lower while returns on Fridays are significantly higher. I verified this claim by applying a one-tail two-sample t -test for equal means on pairs involving Monday and Friday.

For the pairs involving Monday, the null hypothesis is that the average returns are equal, and the alternative hypothesis that the average return on Monday is less than the average return of the day it was compared to. The pairing provided p -values of 0.116699, 0.035023, 0.031487, and 0.031158 when Monday is paired with Tuesday, Wednesday, Thursday, and Friday, respectively. The results indicate that at $\alpha = 0.05$, the null hypothesis for the pairs Monday/Wednesday, Monday/Thursday, and Monday/Friday are rejected, and conclude that the average return on Monday is less than the average return on Wednesday, Thursday, and Friday.

For the pairs involving Friday, the null hypothesis is that the average returns are equal, and the alternative hypothesis is that the average return on Friday is greater than the average return of the day it was compared to. Results show the p -values of 0.031158, 0.00022, 0.490682, and 0.484477 when Friday is paired

with Monday, Tuesday, Wednesday, and Thursday, respectively. The results indicate that at $\alpha = 0.05$, the null hypothesis for the pairs Friday/Monday and Friday/Tuesday is rejected. Therefore, the average return on Friday is greater than the average return on Monday and Tuesday.

Efficient Market Theory

The EMT traces its roots to the random walk model of stock prices which indicates that current stock prices are independent or uncorrelated to previous stock prices. EMT believes that stock prices fully reflect all the information (including but not limited to price histories) about a stock (Cunningham, 2001, p. 23), unlike the noise theory model where trading is based on information that is unrelated to the fundamental asset values (Cunningham, 2001, p. 26). There are three forms of such efficiency—weak form, semi-strong form, and strong form.

In its weak form, a stock price is a reflection only of the information from its price history. The semi-strong form believes a stock price is a function of its price history and all publicly available information about it; while the strong form indicates that stocks prices are a function of all existing information (publicly available or not) about a stock. The strong form of efficiency in the EMT has been discredited by insider trading scandals. It has been observed that possession of nonpublic information has been used to make

abnormally high returns (Cunningham, 2001, p. 25). In short, only the weak form and semi-strong form of EMT are taken into consideration.

Among the three forms of efficiency, it is the weak form that is verifiable from the dataset, and I focused on this aspect of the theory. An analysis for past prices to predict future prices is believed to be useless because any information from the said analysis would already have been included in its current price (Malkiel, 1999, p. 242). Furthermore, if information flows freely, then the changes in tomorrow’s prices will reflect tomorrow’s news only, which makes it independent from the price change today (Malkiel, 1999, p. 243). Hence, the changes in the prices behave like a random walk since the future price changes are random departures from previous prices.

One way of checking out the dependence structure of the process is by looking at the empirical values of its Hurst index. The Hurst index H of a process indicates the type of memory that is embedded in the dataset. It takes values on the interval $(0, 1)$ and a value of $H < 0.5$ indicates that the process has a short memory (mean reverting), while $H = 0.5$ means the process has no memory (random walk). When $H > 0.5$, the process has long-memory (persistence or correlation). It can be interpreted that the value of H is the probability that the next outcome is similar to the present outcome although this value may change as the process progresses (Cunningham, 2001, p. 36).

De Vera and Gabriel (2016, p. 11) have shown that the Hurst index of the dataset of returns from March 1, 1990, to October 28, 2016, were 0.546190, 0.522836, and 0.582895 which were obtained using the Rescaled Range Analysis, Modified Rescaled Range Analysis, and the Detrended Fluctuation Analysis, respectively. By definition, these values indicate the presence of long-memory in the process although these are still close to the value of 0.5 and these values do not indicate strong persistence. At a value of 0.5, the process has no memory, which means that its future cannot be predicted by the past and, in the stock market, this would mean that the short-term changes cannot be predicted (Malkiel, 1999, p. 24).

Discrete-Time Markov Chains

Another way of checking trends in the stock market index is by building Markov chains (Doubleday &

Esunge, 2011). Formally, a discrete-time Markov chain over a countable state space (Ross, 1996, p. 163) can be defined as:

A stochastic process $\{X_n, n = 0,1,2, \dots\}$ on a countable set S is a Markov Chain if, for any i and $j, i, j \in S$ and $n \geq 0$,

$$P\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \dots, X_1 = i_1, X_0 = i_0\} = p_{ij}, \tag{4}$$

for all states $i_0, i_1, \dots, i_{n-1}, i, j$ and all $n \geq 0$.

Equation (4) shows that in a Markov chain the future state X_{n+1} depends only on the present state X_n , given knowledge of the past states X_0, X_1, \dots, X_{n-1} , and the present state X_n . The transition probability p_{ij} is the probability that the Markov chain jumps from state i to state j and these satisfy the following conditions:

$$p_{ij} \geq 0, \quad i, j \geq 0; \quad \sum_{j=0}^{\infty} p_{ij} = 1, \quad i = 0,1, \dots \tag{5}$$

Let \mathbf{P} denote the matrix containing the transition probabilities p_{ij} to have

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \dots \\ p_{10} & p_{11} & p_{12} & \dots \\ \vdots & \vdots & \vdots & \vdots \\ p_{i0} & p_{i1} & p_{i2} & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \tag{6}$$

The Markov chain can be set up based on the basic movements of the daily returns which are negative, unchanged, and positive, denoted by N, 0, and P, respectively. In determining the transition probability matrix for the dataset, I count the number of the following transitions: NN, N0, NP, ON, 00, 0P, PN, P0, and PP. A transition denoted by, say, NP, denotes a change from a negative to a positive return from one trading day to the next. Let n_{ij} denote the number of transitions from i to j , where $i, j \in \{N, 0, P\}$. To obtain the transition probability p_{ij} , take the following ratio:

$$p_{ij} = \frac{n_{ij}}{\sum_{k \in S} n_{ik}}. \tag{7}$$

The transition probability matrix for this Markov chain is given by

$$\mathbf{P} = \begin{matrix} & \begin{matrix} N & 0 & P \end{matrix} \\ \begin{matrix} N \\ 0 \\ P \end{matrix} & \begin{bmatrix} 0.556449 & 0.000309 & 0.443242 \\ 0 & 0 & 1 \\ 0.423958 & 0 & 0.576047 \end{bmatrix} \end{matrix}. \tag{8}$$

In a day to day basis, the matrix shows that the probability of transitioning from a positive return to a positive return is 0.576047 while transitioning from a negative return to a negative return happens with probability 0.556449. The zero entries indicate that transitions 0N, 00, and P0 are not found in the dataset.

In a Markov chain, the probability of transitioning from one state to another in n transitions by evaluating \mathbf{P}^n (multiply the transition probability matrix \mathbf{P} to itself n times) can also be determined. The matrix \mathbf{P}^n contains the n -step transition probabilities $p_{ij}^n = \{X_{m+n} = j | X_m = i\}$ (Ross, 1996, p. 168). In some cases, the value of $\pi_j = \lim_{n \rightarrow \infty} p_{ij}^n$ exists and it is positive, which gives a unique stationary distribution that represents the long-run proportion of time that the chain is in j (Ross, 1996, p. 177).

The Markov chain with transition probability matrix in Equation (8) is

$$\lim_{n \rightarrow \infty} \mathbf{P}^n = \begin{bmatrix} 0.488607 & 0.000151 & 0.511242 \\ 0.488607 & 0.000151 & 0.511242 \\ 0.488607 & 0.000151 & 0.511242 \end{bmatrix} \quad (9)$$

which indicates that, in the long-run, the probability of having a negative, zero, and positive return are $\pi_N = 0.488607$, $\pi_0 = 0.000151$, and $\pi_P = 0.511242$, respectively. In the dataset, a zero return was only obtained once, which is consistent with the very small value of π_0 . If the daily returns follow a random walk, the Markov chain obtained would have shown a 50-50 split between being positive and negative in the long run, but the values obtained are also quite close.

The existence of such stationary probabilities happens when the Markov chain is irreducible, aperiodic, and positive recurrent (Ross, 1996). A Markov chain is irreducible if any state can be reached from any other state, that is, for every pair of states $i, j, \in S$ an $n > 0$ can be found such that $p_{ij}^n > 0$. It is aperiodic when all of its states $i \in S$ has period one. Let the period of the state i be

$$d(i) = \gcd\{n | p_{ii}^n > 0 \text{ for all } n > 0\}, \quad (10)$$

where gcd stands for greatest common divisor. Lastly, a Markov chain is positive recurrent if all of its states $i \in S$ only needs, on average, a finite number of transitions to return to i from state i .

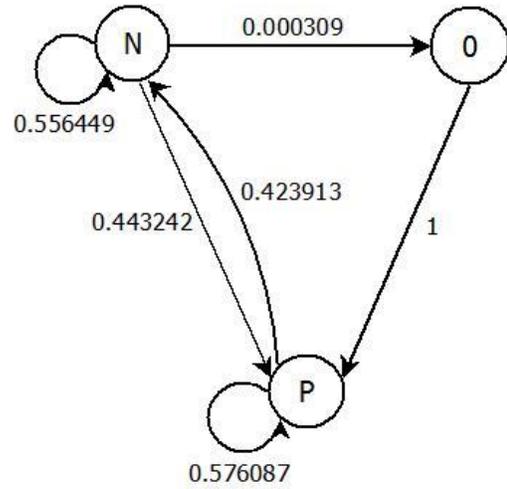


Figure 7. The transition diagram (together with the transition probabilities) associated with the transition probability matrix in Equation (8) shows that all states are accessible from each other ($p_{ij}^n > 0$ for some $n > 0$), that is, the Markov chain can transition to any state from any of the other states.

Figure 7 shows that any state in the Markov chain can be reached from any other state which makes it irreducible. When a chain is irreducible, all its states have the same period; and the states are either all transient or recurrent. It can also be viewed that all the states have period one which makes the chain aperiodic; this is evident by the self-transitions ($p_{pp} > 0$ and $p_{NN} > 0$). In an irreducible finite state Markov chain, all the states have to be positive recurrent for the process not to eventually terminate.

Let μ_{jj} be the expected number of transitions needed to go from state j to j . Alternatively, μ_{jj} can also be viewed as the mean return time to state j (starting from j). In an irreducible, aperiodic Markov chain, $\lim_{n \rightarrow \infty} p_{ij}^n = 1/\mu_{jj}$ (Ross, 1996). In relation to the Markov chain under discussion, I applied this to the results in Equation (9) and obtained $\mu_{NN} = 1/0.488607 = 2.046634$, $\mu_{00} = 1/0.000151 = 6616.769$, and $\mu_{PP} = 1/0.511242 = 1.956021$. The mean return time for states N and P are practically the same which indicates that starting from a positive/negative return, the process (on average) returns to a positive/negative return in roughly two trading days. The value of μ_{00} is quite large as a result of the very small value of π_0 .

The analysis can be expanded using Markov chains by exploring more states such as, in this case,

transitions to a burst or a bust (growth or drop by at least 3%). The state space is now $S = \{\text{bust, neg, 0, pos, burst}\}$, where neg and post denotes a growth or drop of less than 3%. The transition probability matrix of the second Markov chain is

$$\mathbf{P} = \begin{array}{c|ccccc} & \text{bust} & \text{neg} & 0 & \text{pos} & \text{burst} \\ \hline \text{bust} & 0.159236 & 0.407643 & 0 & 0.350318 & 0.082803 \\ \text{neg} & 0.030470 & 0.525769 & 0.000324 & 0.433063 & 0.010373 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \text{pos} & 0.010494 & 0.416049 & 0 & 0.547839 & 0.025617 \\ \text{burst} & 0.025806 & 0.329032 & 0 & 0.470968 & 0.174194 \end{array} \quad (10)$$

It can be seen that from a bust, the next likely state would still be negative (0.407643) while a burst will most likely transition to a positive return (0.470968). This shows that after a depression or elation in stock market prices the market would likely stay in the same depressed or elated state; it is also more persistent during a euphoric state.

This transition probability matrix also converges to a stationary distribution:

$$\lim_{n \rightarrow \infty} \mathbf{P}^n = \begin{array}{c|ccccc} & 0.023652 & 0.464748 & 0.000151 & 0.488099 & 0.023350 \\ \hline & 0.023652 & 0.464748 & 0.000151 & 0.488099 & 0.023350 \\ & 0.023652 & 0.464748 & 0.000151 & 0.488099 & 0.023350 \\ & 0.023652 & 0.464748 & 0.000151 & 0.488099 & 0.023350 \\ & 0.023652 & 0.464748 & 0.000151 & 0.488099 & 0.023350 \end{array} \quad (11)$$

As shown in Equation (9), the long-run probability for 0 is $\pi_0 = 0.000151$. The long-run probabilities of bust and burst are $\pi_{\text{bust}} = 0.023652$ and $\pi_{\text{burst}} = 0.023350$, respectively. For negative and positive returns within 3%, the $\pi_{\text{neg}} = 0.464748$ and $\pi_{\text{pos}} = 0.488099$. In the long-run, there are slightly more positive returns than negative returns but there are slightly more busts than bursts. The mean return times for the states bust and bursts are $\mu_{\text{bust,bust}} = 1/0.023652 = 42.280254$ and $\mu_{\text{burst,burst}} = 1/0.023350 = 42.825806$ trading days, respectively.

Forecasting

The time series of the daily returns can now be examined and do day ahead forecasting with several time domain forecasting methods. The time series analysis for this section of the study was generated using the Real Statistics Resource Pack Software Release 5.6 (Zaiontz, 2018). Let the value at time t be y_t and denote its forecasted value by \hat{y}_t . The error term at time t is denoted by $e_t = y_t - \hat{y}_t$.

Basic Time Series Forecasting

The basic methods are helpful in doing day ahead forecast. The comparison of the basic forecasting methods is based on the values of the mean average error (MAE) $\sum_{t=1}^N |e_t|/N$ and the mean square error (MSE) $\sum_{t=1}^N (e_t)^2/N$, where N is the length of the forecasted series.

For the simple moving average method, take the latest n values of the data to have

$$\hat{y}_t = \frac{y_{t-1} + y_{t-2} + \dots + y_{t-n}}{n} \quad (12)$$

The quantity n is the number of lags and I generated the forecasted series for n from 1 to 30. For the dataset, it was observed that the values of the MAE and MSE decrease as the lag n increases. Although small values of the MAE and MSE are preferred, bear in mind that having a large value of n makes forecasted value insensitive because it smoothens out random variations but is slow in following the real changes as shown in Figure 8 (Waters, 1998, p. 168).

Note that as n increases, the length of the forecasted time series decreases so the decrease in the values of the MSE and MAE as the lag n increases may be partly attributed to the shorter length. To make the values of the MSE and MAE for simple moving average models comparable, I compared only the values of the MSE and MAE from the 31st value. In doing so, the pattern persisted.

Although the simple moving average cannot capture the random noise in the data, it was still useful in forecasting the direction of the change in the data. Being able to predict the signs accurately is useful with the dataset. A moving average model may not be able to identify the amount of increase or decrease adequately but it can identify the signs of the time series accurately, which is useful in predicting whether an increase or decrease is coming. Pesaran and Timmermann (2004) made a study on the usefulness of such information in economics and financial data.

I applied the Pesaran-Timmermann test to find out if the model is adequate in predicting the change in the direction of the time series. This test indicates whether the model can predict the signs accurately and it is not applicable when all the signs of the values in the time series are the same (Zaiontz, 2018, Pesaran-Timmermann, 2004). The test was applied to all the simple moving average models with $n < 30$, and

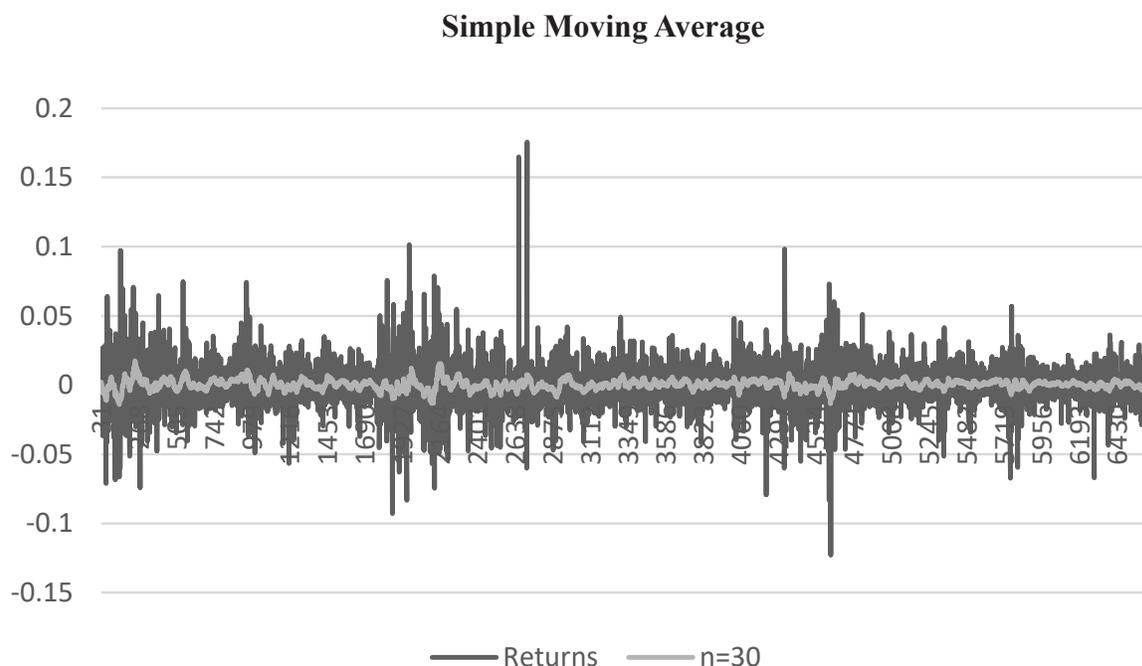


Figure 8. The graph that the Simple Moving Average forecast for $n = 30$ is not able to follow the fluctuations of the actual data although it has the minimum MAE and MSE of 0.010419 and 0.000224, respectively, among simple moving average models with $n \leq 30$.

every model indicated that at the significance level of $\alpha = 0.05$, the null hypothesis that the model does not forecast the sign of the data is rejected. This means that the model can adequately predict the signs of the values in the time series. In fact, the largest p -value obtained when the test was applied to all the simple moving average models is 0.003652.

A simple average model with lag n assumes that the past values have the same contribution to the forecasted value but that assumption can be relaxed and go with a weighted moving average given by

$$\hat{y}_t = w_1 y_{t-1} + w_2 y_{t-2} + \dots + w_n y_{t-n} \quad (13)$$

where w_i denotes the weight of the value i units away from the forecasted value and $\sum_{i=1}^n w_i = 1$. Starting from a lag of $n = 2$, I searched for the first value of the lag n that gives MAE and MSE values that are smaller than the values generated from any of the generated simple moving average models. The first value of n that satisfies the conditions is $n = 15$. In this model, the MAE is 0.010396 while the MSE is 0.000221 and these were taken from the 16th value onwards; and these

values are still smaller than simple moving average models where the MAE and MSE were taken from the 31st value onwards.

In Figure 9, the fluctuation of the dataset is better captured by a weighted moving average model. This weighted moving average model with $n = 15$ has equation

$$\begin{aligned} \hat{y}_t = & 0.204346y_{t-1} + 0.031473y_{t-2} + 0.049132y_{t-3} \\ & + 0.076488y_{t-4} + 0.033993y_{t-5} + 0.038136y_{t-6} \\ & + 0.054568y_{t-7} + 0.075267y_{t-8} + 0.062403y_{t-9} \\ & + 0.044207y_{t-10} + 0.054251y_{t-11} + 0.085812y_{t-12} \\ & + 0.079880y_{t-13} + 0.034844y_{t-14} + 0.075200y_{t-15}. \end{aligned} \quad (14)$$

For all the weighted moving average models generated ($2 \leq n \leq 15$), the value just before the forecasted value has always obtained the largest weight but it is not evident that the weight decreases the further the past value is from the forecasted value. I also performed the Pesaran-Timmermann test on these models and at a significance level of $\alpha = 0.05$ I got the same result as the simple moving average models; in this case the $-$ values have been rounded off to zero.

Another way of relaxing the assumption of having the past n values the same weight is by employing a



Figure 9. The weighted moving average with $n = 15$ gave values of the MAE and MSE that are smaller than those values from the simple moving average models. It can be seen that its fluctuations in the forecasted model are more in tune with the dataset.

simple exponential smoothing. A simple exponential model is defined as

$$\hat{y}_1 = y_1 \quad \hat{y}_{t+1} = \alpha y_t + (1 - \alpha)\hat{y}_t, \quad (15)$$

where $t \geq 1$ and $\alpha \in (0,1)$ are referred to as the smoothing constant. The \hat{y}_{t+1} can be rewritten in terms of the values in the given time series only as:

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha)y_{t-1} + (1 - \alpha)^2 y_{t-2} + \dots + (1 - \alpha)^{t-2} y_2 + (1 - \alpha)^{t-1} y_1. \quad (16)$$

When applied to the dataset, the simple exponential smoothing gave the model $\hat{y}_t = 0.001737y_t + 0.998263\hat{y}_t$, that gave the smallest MSE of 0.000218 while the simple exponential model that gave the smallest MAE of 0.010238 is $\hat{y}_t = 0.0002180y_t + 0.997190\hat{y}_t$. Although these models gave the smallest MAE and MSE so far, these models were not able to follow the fluctuations of the dataset. The model has essentially tuned out the random noise in the stock market. When the Pesaran-Timmermann test was applied to both models and at a significance level of $\alpha = 0.05$, the null hypothesis is rejected. These models can predict the sign on the time series accurately.

Among the basic forecasting method applied to the dataset, it is the weighted moving average with a lag of 15 that has the right balance between a small MAE and MSE and in capturing the inherent fluctuations in the dataset.

Autoregressive Moving Average (ARMA) Processes

Before discussing with ARMA processes, I considered autoregressive (AR) and moving average (MA) processes and looked at the autocorrelations and partial autocorrelations of the data to determine the appropriate order p and q of an $AR(p)$ and $MA(q)$ processes, respectively. An $AR(p)$ process is like the WMA model in the sense that a linear function of the past p values is used to predict the next value with no restriction on the sum of the coefficients of the past values, that is,

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t, \quad (17)$$

where ϕ_0 is a constant and the error terms ε_t are independent and identically distributed (Zaiontz, 2018). The error terms are normally distributed with mean zero and variance σ^2 , that is, $\varepsilon_t \sim N(0, \sigma^2)$. On the other hand, an $MA(q)$ process utilizes a linear function of past errors to predict the next value,

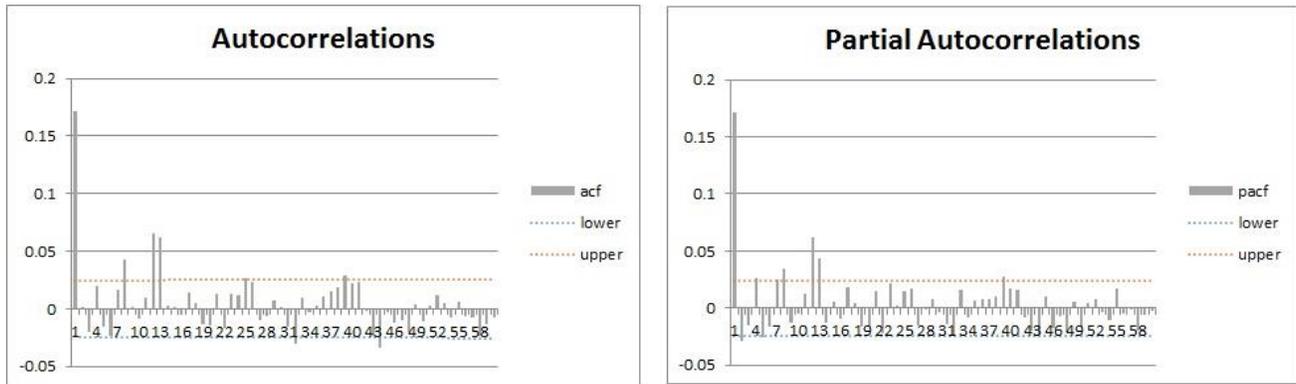


Figure 10. In both the ACF and PACF plots, the values at lags of 1, 9, 12, and 13 are statistically not equal to zero at the significance level of $\alpha = 0.05$. To identify the appropriate AR and MA processes for the data, AR and MA processes were generated from 1 to 13 and saw which performed well based on the Akaike information criterion (AIC) and Bayesian information criterion (BIC).

$$y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q}, \quad (18)$$

where μ is the mean of the process and $\varepsilon_t \sim N(0, \sigma^2)$ are independent (Zaiontz, 2018).

The autocorrelation $ACF(k)$ gives the degree of linear relationship between observations that are units apart while the partial correlation $PACF(k)$ is the autocorrelation between observations that are also k units apart without taking into consideration the values between the observations. The plot of the ACF also indicates whether the data is stationary or not; a stationary series shows a plot where the values exponentially decay.

In Figure 10, the ACF cuts off after lag one although there are lags where the values are not statistically equal to zero at the significance level of $\alpha = 0.05$. This indicates that the data is most likely stationary.

I applied the Augmented Dickey-Fuller (ADF) test to determine whether the series is stationary or not. When a series is stationary, its mean, variance, and covariance are constant. The ADF test takes into consideration three types of time series—no constant and trend, with constant no trend, and with constant and trend. This test takes the null hypothesis that the series is not stationary. In Figures 8 and 9, the data do not have a trend but I still applied each type of test at a significance level of $\alpha = 0.05$ and results all pointed to a stationary series. This means that no differencing is necessary on the given dataset to obtain a stationary time series.

The appropriate order of the fitted AR process can be chosen based on the PACF (Maddala, 2001) while using the ACF to determine the correct order of the fitted $MA(q)$ process because the ACF for all $k > q$ is zero. Figure 10 shows the ACF and PACF plots of the data and, for both plots, the values with lags 1, 9, 12, and 13 indicate that the values are not statistically equal to zero at the significance level of $\alpha = 0.05$. This indicates that there is persistence or non-zero correlation for values that are two weeks apart. Although the plot of the PACF drops after the first lag, I still fitted $AR(p)$ processes where $1 \leq p \leq 13$.

The choice for the best fitted AR process is done through the Akaike information criterion (AIC) and Bayesian information criterion (BIC). I chose the process with the lowest AIC or BIC (Maddala, 2001, p. 527). The AIC and BIC are computed as $N \ln[\sum_{t=1}^N (y_t - \hat{y}_t)^2 / N] + 2k$ and $N \ln[\sum_{t=1}^N (y_t - \hat{y}_t)^2 / N] + k \ln N$, respectively, where N is the length of the series and $k = i = q + 1$ is the number of parameters in the model with no constant (Zaiontz, 2018). Observe that the values of the AIC and BIC increases with the number of model parameters k and the only way for the values of the AIC and BIC to decrease while k increases are for the sum of squares error (SSE) $\sum_{t=1}^N (y_t - \hat{y}_t)^2$ to substantially decrease to compensate for the increase in k .

I started by fitting AR processes with constant included and observed that for all the processes, except for $AR(6)$, the constant is statistically equal to zero at the significance level of $\alpha = 0.05$. Comparing the AIC

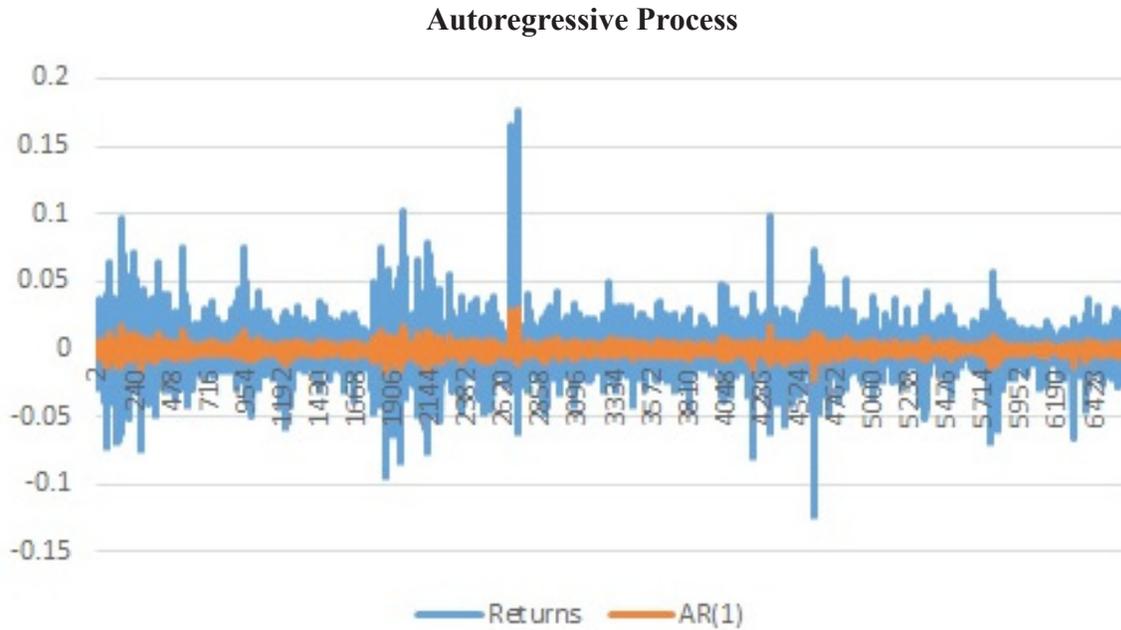


Figure 11. The plot of the fitted AR(1) process with no constant looks similar to the fitted WMA with 15 lags although this process has a less number of parameters, and although more data points are included, the fitted AR(1) process has an MSE of 0.000212 which is lower than the MSE of 0.000221 of the fitted WMA model.

and BIC for the fitted AR processes with and with no constant and AR(6) with a constant, it was the AR(1) with no constant that gave the least values for both the AIC and BIC. These implies that the increase in both values with the number of parameters is more dominant than the change in the SSE.

The equation of the AR(1) with no constant is $y_t = 0.172044y_{t-1} + \varepsilon_t$. The parameter of this model is statistically not equal to zero at the significance level of $\alpha = 0.05$. The Pesaran-Timmermann test also indicates that the fitted model can predict the signs adequately. I applied the Ljung-Box test to the residuals ε_t to verify that the residuals are indeed random. For this test, the null hypothesis is that the residuals are random. At the significance level of $\alpha = 0.05$, the value of p -value is 0.719167 which is greater than α , thus the null hypothesis is accepted. This means that the AR(1) process with no constant provides an adequate fit.

I compared the forecasts of the fitted AR(1) process and the fitted weighted moving average model with $n = 15$ via the Diebold-Mariano (DM) test. The DM test makes assumptions on the forecast error loss differential given by $d_{12t} = \varepsilon_{1t}^2 - \varepsilon_{2t}^2$, where ε_{1t} and ε_{2t} are the forecast error of the first and second fitted models, respectively, at time t (Diebold, 2012, p. 2).

In Zaiontz (2018), the DM test also considers the loss differential given by $d_{12t} = |\varepsilon_{1t}| - |\varepsilon_{2t}|$ and the key assumption on the DM test is that d_{12t} is stationary. For the fitted models, the plots of the ACF decays exponentially for both loss differential functions, which indicates that both series are stationary as shown in Figure 12. I also applied the ADF test to both loss differentials and both series came out stationary for all the types considered by the test at $\alpha = 0.05$. When the DM test was applied at $\alpha = 0.05$ the p -values round off to zero for both loss differentials of square errors and absolute errors. This means that both models have different forecasting abilities.

To improve the fitted AR(1) process, I added some moving average terms to the model and came up with a fitted autoregressive moving average (ARMA) model; the general equation of an ARMA(p, q) process is

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}. \tag{19}$$

I employed the AIC and BIC in picking the best ARMA process. The ACF plot of the daily returns shows that after the first lag, most values are statistically zero at the significance level of $\alpha = 0.05$. I looked at ARMA(1, q) processes where $1 < q < 6$ and

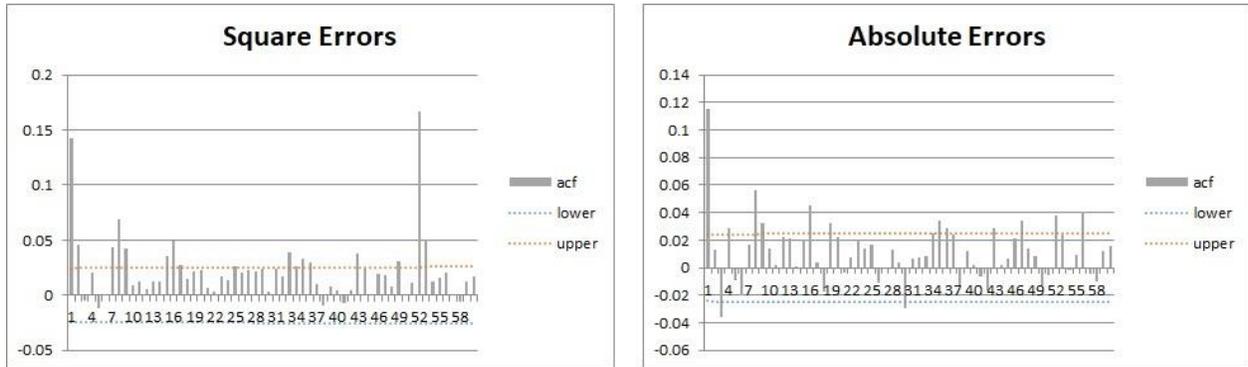


Figure 12. The plots of the ACF for the loss differential of square errors and absolute errors show exponential decay which indicates the stationarity of the series. To confirm this result, I also applied the ADF test and obtained the same result at $\alpha = 0.05$.

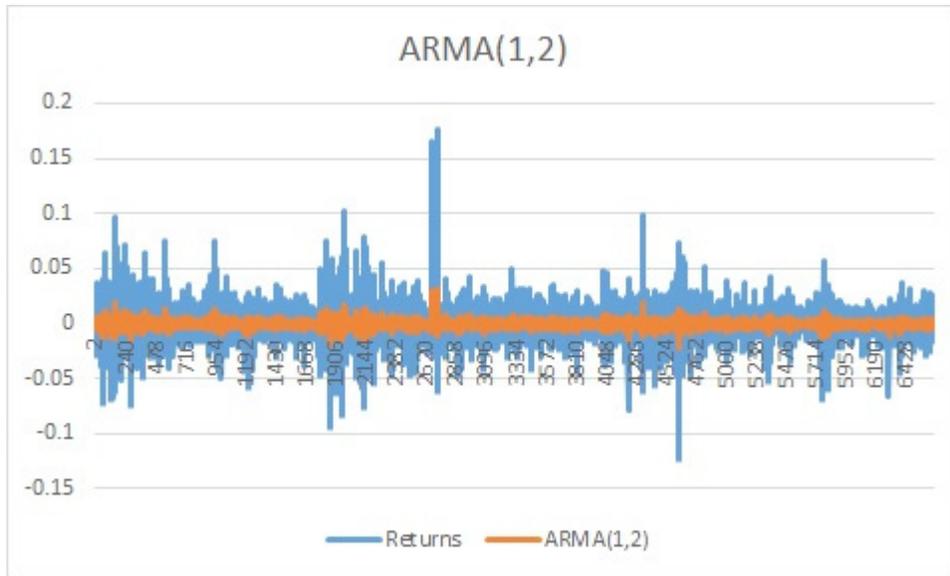


Figure 13. The MSE for this model also rounds off to 0.000211 which is similar to the MSE from the fitted AR(1) process. When the DM test was applied, the result indicates the models’ forecasts accuracy are the same.

compare the values of the AIC and BIC. The value of the BIC increased as the number of parameters decreased while the AIC is minimum at $q = 3$ but the values of this model are all statistically equal to zero at $\alpha = 0.05$. Because of this, I picked an ARMA(1,2) where all the parameters are not statistically equal to zero at $\alpha = 0.05$. The fitted process is given by

$$y_t = -0.72242y_{t-1} + \varepsilon_t + 0.901773\varepsilon_{t-1} + 0.148019\varepsilon_{t-2}. \quad (20)$$

I applied the Ljung-Box test to the residuals to verify that the residuals are indeed random. The p -value is 0.830033 so at $\alpha = 0.05$ the null hypothesis that the residuals are random is accepted.

When the Pesaran-Timmermann test was applied, the result indicates that the model can predict the signs accurately. The DM test was applied to AR(1) and AR(1,2) and the p -values obtained were all greater than $\alpha = 0.05$ which indicate that forecasts accuracy of the processes is the same.

Summary and Conclusion

The anomalies observed in other stock markets also exist in the local market. By having a good average effective rate of returns in December and January (as shown in Figure 3), being invested in the stock market only in those two months of each year maybe be a

profitable investing strategy. Out of the 26 December–January pairs, the effective rate of returns from both months were positive 11 times while only twice were the returns negative from both months. Furthermore, the effective rate of return on January was greater than the effective rate of return on December in 16 of those pairs. Figure 4 also suggests that, on average, the best trading day to buy is on Tuesday and to sell on Thursday although the differences of the averages in the last three trading days is quite small and an ANOVA supports the observation that the averages are equal. There seems to be a difference between the averages of the first two days of the week but it is not statistically significant as shown by a two-sample *t*-test for equal means.

The Markov chain with states negative, unchanged, and positive showed that, in the long run, 51.1242% of the time the returns are positive and 48.8607% of the time the returns are negative. The probability of transitioning from a positive return to a positive return is 0.576047 while transitioning from a negative return to a negative return has a probability of 0.556449. These transition probabilities are quite close to the empirical values of the Hurst index which varied from 0.522836 to 0.582895 and, as mentioned earlier, these values indicate the probability that the current behavior will persist.

Although transitioning from a positive trading day to a positive trading day and a negative trading day to a negative trading day has a probability that is above 0.5, this does not indicate a strong persistence. It could just as easily change to something closer to 0.5 the moment more values were incorporated into the dataset which will then indicate that the time series behaves like a random walk. The behavior of the time series changes as we go along and it is important to have an idea of where it is possibly going.

When the bust and burst states were introduced, in the long run, 2.3652% and 2.335% of the returns are busts and bursts, respectively. It would be interesting to look at the political and economic events that have triggered such erratic behavior.

I also looked into day ahead forecasting based time domain models and an AR(1) and ARMA(1,2) for this data has the same forecasting ability. I did not apply an autoregressive integrated moving average model because the data of daily returns is a stationary series. I recommend considering frequency domain

forecasting methods to model the random noise in the data. Although the models were unable to predict random noise, they were able to accurately predict the signs of the values. The PSE index is composed of 30 stocks and having the ability to accurately predict the overall behavior of these 30 stocks on a day-to-day basis is a challenge.

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